Predictor Corrector Controller using Wiener Fuzzy Convolution Model

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Abstract

This paper investigates the application of hybrid fuzzy models in modelling and model based predictive control of a delayed and distributed parameter system with a nonlinear feature.

The presented hybrid fuzzy convolution model consists of a nonlinear fuzzy steady-state model and an impulse response model based dynamic part. The proposed non-linear block-oriented dynamic model is applied to form a predictor corrector controller.

The control of a laboratory-sized heating system is chosen as a realistic nonlinear case study for the demonstration of the control algorithm. The proposed model based controller is shown to be capable of controlling the nonlinear process that operates over wide range.

1. Introduction

The concept of model based predictive control (MBPC) has been heralded as one of the most significant control developments in recent years [1]. The wide range of choice of model structures, prediction horizon, and optimisation criteria allows the designer to easily tailor MBPC to his application.

The choice of the optimisation criteria and the adequate model structure has a distinguished role in the control design procedure of these controllers. Control theory has been extensively developed for linear systems, yet linear model based control strategies have been shown to perform poorly when applied to certain non-linear systems. The introduction of non-linear models within the MPC algorithm is one of the most challenging tasks of our days [2]. Unfortunately, first-principle models are often too complex and often there is not enough *a priori* knowledge to design accurate, usable models. However, most of the advanced techniques require models of restricted complexity.

In contrast to conventional models, fuzzy models can also represent highly nonlinear processes and can smoothly integrate *a priori* knowledge with information obtained from process data [3]. Therefore, fuzzy models were also incorporated into the MBPC scheme recently [4, 5, 6]. These solutions are often too complex to make fast real-word implementation because of the problems of fuzzy model identification and the realisation of the control algorithm.

In order to solve these problems, the proposed modelling approach assumes that the process can adequately be modelled using the combination of a non-linear gain and a gain-independent dynamic part, similar to Wiener and Hammerstein models [7, 8]. This assumption is supported by the fact that such block-oriented model structures are able to tackle non-linear effects encountered in most chemical processes such as distillation, and pH neutralisation as well as in furnaces and reactors [9, 10].

The other contribution of the paper is the introduction of a predictor corrector controller for this type of hybrid convolution models. Because of the special structure of the proposed hybrid fuzzy convolution model (HFCM), the implementation of the predictor corrector control algorithm is very straightforward in contrast to several other MBPC algorithms [11,12].

The control of a laboratory-sized heating system is chosen as a case study for demonstration.

The paper is organised as follows: The description of the studied system is in section 2, section 3 deals with the hybrid fuzzy model of the system. The controller structure and the results and discussion are presented in section 4 and 5 respectively. Section 6 offers the relevant conclusions.

2. The Fuzzy Convolution Process Model

Fuzzy modelling and control has been widely applied to many chemical engineering problems [13]. In fact, it can be used as a means of both capturing humans' expert knowledge and dealing with uncertainty. The fuzzy process models can be initialised by expert knowledge and can be adapted by the use of process data [3].

The Nonlinear AutoRegressive with eXogenous input NARX model is frequently used with many nonlinear identification methods, such as neural networks and fuzzy models, for dynamic modelling of chemical process systems [14]. As with all system identification strategies, this type of NARX modeling has several weaknesses. Problems associated with system identification in high dimensions are explained by the course of dimensionality [15]. With this growing, the implementation costs and the costs of calculating an output grow exponentially, making the modelling of a high order and distributed parameter systems infeasible.

It has been shown that non-linear effects encountered in most chemical processes, distillation columns, pH neutralisation processes, heat-exchangers, etc. can be effectively modelled as the combination of a non-linear static element and a gain independent dynamic part [2].

Consequently, the physical system should be expressed as the combination of a steady-state and a dynamic part.

The proposed block-oriented hybrid fuzzy convolution model is a Wiener convolution model. The output of the non-linear dynamic model can be described as:

$$y_m(k+j) = f\left(\sum_{i=1}^{N} g_i \cdot u(k-i+j), x_2, \ldots, x_n\right) \tag{1}$$

where the convolution has to handle the process gain independent g_i impulse response model and the past input values, $u(k-i+1)$, over the N model horizon. The nonlinearity is introduced by a fuzzy model that represents the steady-state behaviour of the process:

$$y_s = f(u_s, x_2, \ldots, x_n) \tag{2}$$

where the steady-state output depends on u_s steady state input and other operating parameters having effects on y_s.

2.1 The Steady-state part, Fuzzy Model

The steady-state behaviour of the system is described by zero-order Takagi-Sugeno [16] fuzzy models formulated with a set of rules as follows:

$$r_{i_1,\ldots,i_n}: \text{ if } x_1 \text{ is } A_{1,i_1} \text{ and ... and } x_n \text{ is } A_{n,i_n} \text{ then } y_s = d_{i_1,\ldots,i_n} \tag{3}$$

where $r_{i_1,\ldots,i_n}$ denotes the fuzzy implication, n is the number of inputs, and $\mathbf{x} = [x_1,.., x_n]^T$ is an n dimension vector containing all inputs of the fuzzy model. $A_{j,i_j}(x_j)$ is the $i_j = 1,2,..., M_j$ -th antecedent fuzzy set referring to the j - th input, whose membership functions are denoted by the same symbols as the fuzzy values, where M_j is the number of the fuzzy sets on the j -th input domain.

Let the first element of the input vector be the steady-state input, $x_1 = u_s$ and $x_2,.., x_n$ the operating parameters having effects on the steady state output. At a given input vector, ***x***, the final output of the fuzzy model is inferred by taking the weighted average of the rule consequences, $d_{i_1,\ldots,i_n}$:

$$y_s = \frac{\sum_{i_1=1}^{M_1} \cdots \sum_{i_n=1}^{M_n} \beta_{i_1,..,i_n} d_{i_1,..,i_n}}{\sum_{i_1=1}^{M_1} \cdots \sum_{i_n=1}^{M_n} \beta_{i_1,..,i_n}} \tag{4}$$

where the weights, $\beta_{i_1,..,i_N} > 0$, implies the overall truth value of the $i_1,..,i_n$ –th rule calculated as:

$$\beta_{i_1,..,i_n} = \prod_{j=1}^{n} A_{j,i_j}(x_j) \tag{5}$$

The triangular membership functions were employed for each fuzzy linguistic value as shown in Fig. 1, where a_{j,i_j} denotes the cores of fuzzy set A_{j,i_j} :

$$a_{j,i_j} = core(A_{j,i_j}) = \left\{ x_j \middle| A_{j,i_j}(x_j) = 1 \right\} \tag{6}$$

Thus, the membership functions are defined as follows:

$$\begin{aligned} A_{j,i_j}(x_j) &= \frac{x_j - a_{j,i_j-1}}{a_{j,i_j} - a_{j,i_j-1}}, \quad a_{j,i_j-1} \le x_j < a_{j,i_j} \\ A_{j,i_j}(x_j) &= \frac{a_{j,i_j+1} - x_j}{a_{j,i_j+1} - a_{j,i_j}}, \quad a_{j,i_j} \le x_j < a_{j,i_j+1} \end{aligned} \tag{7}$$

For the application of the fuzzy model the mapping between $u_s = f_s^{-1}(y_s, x_2,..., x_n)$ has to also be determined. This means the problem of the inversion of the fuzzy model. In this study, the inversion method proposed by Babuska et al. is applied [3].

2.2 The Dynamic Part (The Impulse Response Model)

The identification of the dynamic part of a block-oriented model is a challenging task. In this study an impulse response model (IRM) is applied to represent the dynamic behaviour of the process. In practice, the identification of the parameters of the IRM may be troublesome due to their large number. In some cases this problem can be simplified if the modeller has *a priori* knowledge about the dynamic behaviour of the system. This is especially true for some chemical processes, when the impulse response model relates to the resident time distribution of the operating unit [17].

The residence time of a cascade consisting of continuous perfectly mixed operating units is often used to approximate the behaviour of a partially known process, e.g. distributed parameter system, in chemical engineering practice.

The density function of the residence time distribution of this "general" process can be described as:

$$\varphi(t) = \frac{n_c}{\tau} \cdot \frac{\left(n_c \cdot \frac{t}{\tau}\right)^{n_c - 1}}{(n_c - 1)\,!} \cdot \exp\left(-n_c \cdot \frac{t}{\tau}\right) \tag{8}$$

where n_c is the number of the elements of the cascade and τ is the residence time.

After the identification of the number of the units, n_c, and the average resident time, τ, the discrete impulse response (IRM) model can be easily calculated:

$$g_i = \frac{\varphi(i \cdot \Delta t)}{\sum_{i=1}^{N} \varphi(i \cdot \Delta t)} \tag{9}$$

where Δt denotes the sampling time, i the ith discrete time-step, and N is the model horizon.

3. The Hybrid Fuzzy Convolution Model Based PCC Control

The predictor-corrector principle implies two main components, which act simultaneously. These are the *prediction* of the process output, based on the process model, and the *correction* of the model parameters on the basis of measured data after the control action [11].

The algorithm starts from the fact that at time k, the $y(k)$ value of the controlled variable and the set-point over the prediction horizon (p) are known. Thus, the value of $u(k)$ to be applied in the interval [k, k+1] can be determined by using a certain prediction rule. After applying the control output, the $y(k+1)$ value of the predicted output can be also determined. The realized control action is used to calculate the modelled process output, which is employed in computing the model error that is used to increase the model accuracy.

The control and the correction rule define the PCC. The control rule is formulated as:

$$w(k+p) \equiv y(k+p) = f\left(\sum_{i=p+1}^{N} g_i \cdot u(k-i+j) + \sum_{i=1}^{p} g_i \cdot u(k) + \varepsilon_u(k), x_2, ..., x_n \right) \tag{10}$$

where *w(k+p)*, denotes the required set-point at the *k+p*th discrete time instant and *p* denotes the prediction horizon and $\varepsilon_u(k)$ denotes the controller output error calculated based on the measured process output, *y(k)*, and the correction rule:

$$\varepsilon_u(k) = f^{-1}(y(k), x_2, ..., x_n) - \sum_{i=1}^{N} g_i \cdot u(k-i) \tag{11}$$

The control action of PCC at the *k*th sampling instant can be obtained from equation (10):

$$u(k) = \frac{f^{-1}(w(k+p), x_2, ..., x_n) - \left[\sum_{i=p+1}^{N} g_i u(k+p-i) + \varepsilon_u(k) \right]}{\sum_{i=1}^{p} g_i} \tag{12}$$

Substituting $\varepsilon_u(k)$ in equation (12) the following relation is obtained:

$$u(k) = \frac{f^{-1}(w(k+p), x_2, ..., x_n) - f^{-1}(y(k), x_2, ..., x_n)}{\sum_{i=1}^{p} g_i} + \frac{\sum_{i=1}^{N} g_i u(k-i) - \sum_{i=p+1}^{N} g_i u(k+p-i)}{\sum_{i=1}^{p} g_i} \tag{13}$$

Formally, this is a special proportional controller with bias, where the gain and the bias vary in time. Its usability assumes that the impulse response model (g_i), the past control moves ($u(k-i)$, $i = 1, 2, \ldots N$) and the (u_s, y_s) steady state input-output pairs are known on the whole operating interval.

The PCC controller has two parameters. These are the model horizon, *N*, and the prediction horizon, *p*. The model horizon has to be equal with the settling time of the process. The prediction horizon *p* represents the time at which the controlled variable has to be equal - on the base of the control rule - with the set-point.

4. Simulation Example: Application to Water-heater Process

4.1 The Process Examined

The studied physical system is a laboratory-sized heating system, where the thermal-agent passes through a pair of metal pipes which contain a cartridge heater. The heaters are linked parallel with each other and have a performance of 1 kW. The physical system can be dismantled into three main elements, which are in strong interaction with each other. These elements are the cartridge-heater (CH), the streaming thermal agent (w), and the wall of the pipe (W). A further element is the environment (env). Considering these elements three heat balances can be obtained:

$$V_{CH} \cdot \rho_{CH} \cdot C_{p\,CH} \cdot \frac{\partial T_{CH}}{\partial t}(t,z) = Q(u) - \alpha_1 \cdot A_1 \cdot (T_{CH} - T_w)$$

$$V_w \cdot \rho_w \cdot C_{p_w} \cdot \frac{\partial T_w}{\partial t}(t,z) + \left(F \cdot \rho \cdot C_p\right)_w \cdot \frac{\partial T_w}{\partial z}(t,z) = \alpha_1 \cdot A_1 \cdot (T_{CH} - T_w) - \alpha_2 \cdot A_2 \cdot (T_w - T_W) \qquad (14)$$

$$V_W \cdot \rho_W \cdot C_{p_W} \cdot \frac{\partial T_W}{\partial t}(t,z) = \alpha_2 \cdot A_2 \cdot (T_w - T_W) - \alpha_{env} \cdot A_{env} \cdot (T_W - T_{env})$$

where $z \in [0, L]$, where L is the lengths of the pipe and *Q(u)* is the sinusoidal performance of CH formulated as

$$Q = Q_M \cdot \left[u - \frac{\sin(2\pi \cdot u)}{2\pi} \right] \qquad (15)$$

where Q_M is the maximal performance of the cartridge heater, and u is the heating control signal in electrical voltage.

The description and the values of the parameters used in this study are given in Tab. 1.

The output temperature of the thermal agent $T_{out} = T_w(t, z = L)$ is maintained in desired value by adjusting the heating control signal, u.

The detailed description of the process and its first principle model can be found in [2].

The steady state output of the process, $y_s = T_{out}$ depends only on the water flow-rate, F, and the steady state input, u_s. The fuzzy model can easily represent this relationship:

$$r_{i_1,i_2}: \text{ if } F \text{ is } A_{1,i_1} \text{ and } u_s \text{ is } A_{2,i_2} \text{ then } y_s = d_{i_1,i_2} \qquad (16)$$

For a good modelling performance 6 antecedent fuzzy sets on each input universes were utilised.

Fig. 2. shows the surface representation of the steady-state model fuzzy model.

The fuzzy model was identified by the least-squares error method [3] based on steady-state data pairs on range shown in Fig. 2.

At the shape examining of the impulse response curves generated from the process [2], it has turned out that they are formally analogous with the density function of the residence time distribution of a cascade consisting of CSTRs.

The fitting of equations (9, 10) to measured impulse-response curves at different flow-rates leads to the choice of $n_c = 2$ on the whole operating interval. Thus, g_i - which is in fact the normalised density function of the resident time distribution- can be expressed as:

$$g_i = \frac{q^2 \cdot i \cdot \Delta t \cdot \exp(-q \cdot i \cdot \Delta t)}{\sum_{i=1}^{N} q^2 \cdot i \cdot \Delta t \cdot \exp(-q \cdot i \cdot \Delta t)} \qquad (17)$$

where q is a function of the flow-rate F and Δt =2 sec denotes the sampling time, i the ith discrete time-step, and N the number of the time steps (model horizon). The parameter q is calculated considering that it is a function of the flow-rate. The relation q-F is properly approximated by the fitted equation (Fig. 3) [2]:

$$q = 0.064 \cdot \left(\frac{F}{150} \right)^{0.11} \qquad (18)$$

4.3 Control results

In order to make a comparison of the studied controllers the integral of the square errors (ISE) criteria was used as performance index:

$$ISE = \sum_{k=1}^{I} e(k)^2 + \lambda \cdot \sum_{i=1}^{I} (u(k) - u(k-1))^2 \qquad (19)$$

where $e(k) = w(k) - y(k)$ is the controller error in the kth discrete time step, $I = \frac{t_{\max}}{\Delta t}$ the maximal number of time steps and Δt =2 sec is the sampling time.

The error criterion employs a punishment member for the variation of the manipulated variable, weighted with λ =*5*.

4.3.1 PID control

In spite of the fact that PID controllers meet several difficulties when employing them in the control of constrained systems with high nonlinearity, 80-90 % of all control problems can be executed elegantly with this procedure [18]. Because of their frequent use, it was considered that they serve as a proper base at the comparison and examination of model based predictive controllers.

The parameters of the optimal PID controller were determined by constrained optimisation with the Sequential Quadratic Programming method. Thus, the obtained optimised PID parameters are considered to lead to a superior performance than those obtained by the classical tuning methods (Cohen-Coon, ITAE, etc.). The control task (the series of set-point changes) and the performance of the optimal PID controller (*ISE=114.7*) is shown in Fig 4.

4.3.2 Tuning and Control with Hybrid Fuzzy Convolution Model Based PCC

The PCC controller has two parameters. These are the model horizon (N) and the prediction horizon (p). Fig. 5 shows the effect of the prediction horizon on the performance of the controller under noise-free conditions. The output response becomes less oscillatory and more sluggish as the prediction horizon, p, is increased. It can be seen that the optimum performance is achieved with a prediction horizon of six steps. The performance of this controller is shown in Fig 6.

As the contrast between Fig. 4 and Fig. 6 shows, the hybrid fuzzy convolution model based predictive controller gives better performance (*ISE=19.83*) than the optimal PID controller (*ISE=114.7*). The main reason of this significant improvement is the fact that in the PCC control the future set-point is assumed to be known in advance. If the future set-point trajectory is not known the performance of the proposed controller is *ISE= 94.81* which is still better then the optimised PID controller.

As Fig. 7. Shows the proposed controller is capable of providing good control performance in the case of wide-range set-point changes and load-type disturbances on the water flow-rate.

6. Conclusion

This paper presented the capabilities of the application of fuzzy convolution models in modelling and model based predictive control of a delayed and distributed parameter system with a nonlinear feature. The modelling was realised in order to carry out an adequate model predictive control (MPC). A simple nonlinear hybrid model, which consists of a nonlinear fuzzy steady state model and an impulse response model based dynamic part was designed. The proposed fuzzy convolution model was applied to form a predictor corrector controller. The control of a laboratory-sized heating system was chosen as realistic nonlinear case study for the demonstration of the proposed control algorithm. The proposed fuzzy convolution model based controller was shown to be capable of controlling the nonlinear process that operates over wide range and providing better overall system performance than the optimal PID controller.

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Symbols

g_i	parameters of the impulse response model
N	model horizon
K	steady-state gain
u_s	steady-state input
y_s	steady-state output
$y(k)$	current measured process output
$\varphi(t)$	density function of the residence time distribution
n_c	number of the elements of the cascade
τ	average residence time
$r_{i_1,\ldots,i_n}$	index of the fuzzy rule
n	number of the inputs of the fuzzy model
$\boldsymbol{x} = [x_1,..,x_n]^T$	n vector containing all inputs of the fuzzy model
M_j	number of the fuzzy sets on the j -th input domain
$A_{j,i_j}(x_j)$	$i_j = 1,2,\ldots,M_j$ -th antecedent fuzzy set referring to the j-th input
a_{j,i_j}	cores of the A_{j,i_j} fuzzy set
$d_{i_1,\ldots,i_n}$	consequence parameter of the $i_1,\ldots,i_n$ -th rule
$\beta_{i_1,\ldots,i_n}$	weights of the $i_1,\ldots,i_n$ -th rule

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Table 1. The description and the values of the parameters used in the simulation model of the laboratory sized heating system

Parameter	*Description*	*Nominal value*
L	Length of the pipe	2×480 10^{-3}m
ρ_{CH}	Density of the cartridge	3650 kg/m^3
C_{pCH}	Heat capacity of the cartridge	1047 J/kg·K
A_{CH}	Surface of the cartridge	$2.41 \cdot 10^{-2} m^2$
V_{CH}	Volume of the cartridge	$4.82 \cdot 10^{-5} m^3$
α_1	*CH-w* heat transfer coefficient	$316.3 \ Wm^{-2}K^{-1}$
ρ_w	Density of the water	1000 kg/m^3
C_{pw}	Heat capacity of the water	4186 J/kg·K
t_{in}	Inlet water temperature	11.8 ° C
V_w	Volume of the water	$1.16 \cdot 10^{-4} m^3$
α_2	*w-W* heat transfer coefficient	$1196.1 \ Wm^{-2}K^{-1}$
ρ_W	Density of the wall	7850 kg/m^3
C_{pW}	Heat capacity of the wall	502 J/kg·K
t_{env}	The temperature of the environment	21.6 °C
A_W	Inner surface of the wall	$4.46 \cdot 10^{-2} m^2$
V_W	Volume of the wall	$7.37 \cdot 10^{-5} m^3$
A_{env}	Outer surface of the wall	$5.36 \cdot 10^{-2} m^2$
α_{env}	*W-env* heat transfer coefficient	$1015.9 \ Wm^{-2}K^{-1}$

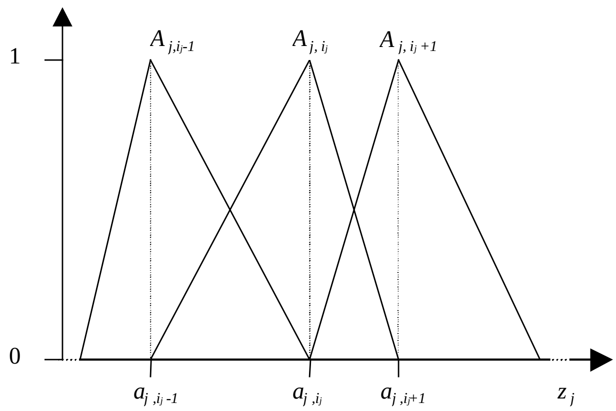

Figure 1. Membership functions used for the fuzzy model

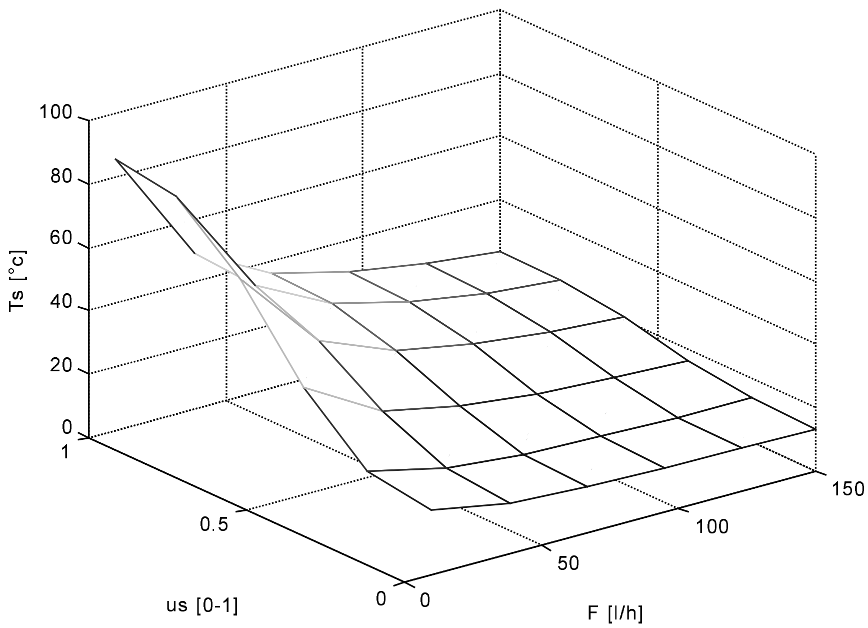

Figure 2. The surface representation of the steady-state process model

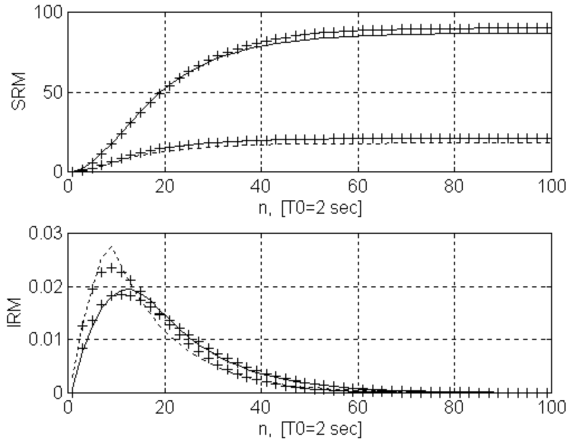

Figure 3. SRMs and IRMs of the process and the fitted model

(— real system at 25 l/h, - - - real system at 150 l/h, +++ fitted model)

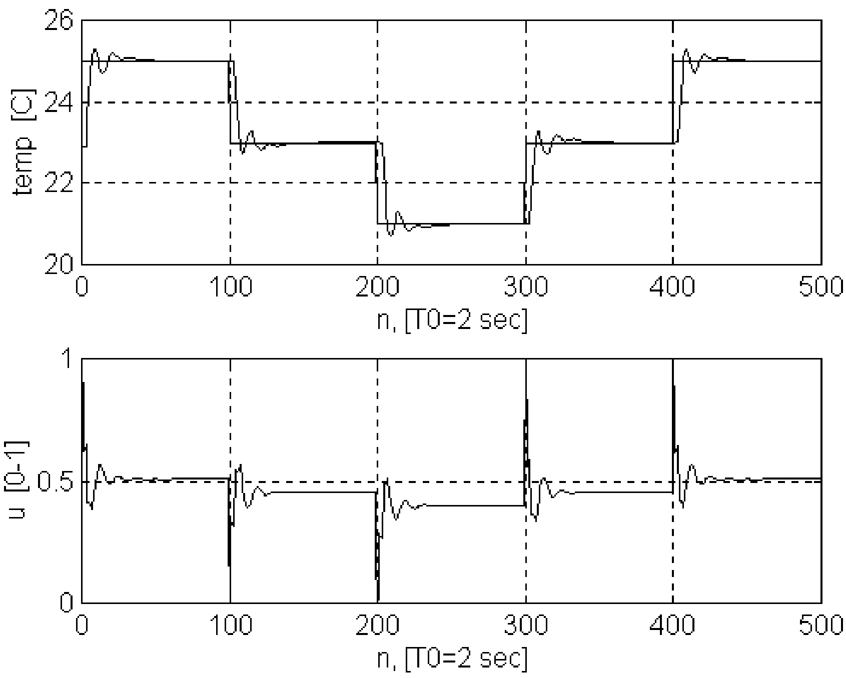

Figure 4. The performance of the optimal PID controller

(P = 0.076, I = 24.7, D = 3.7, χ = 5, **ISE=114.7**)

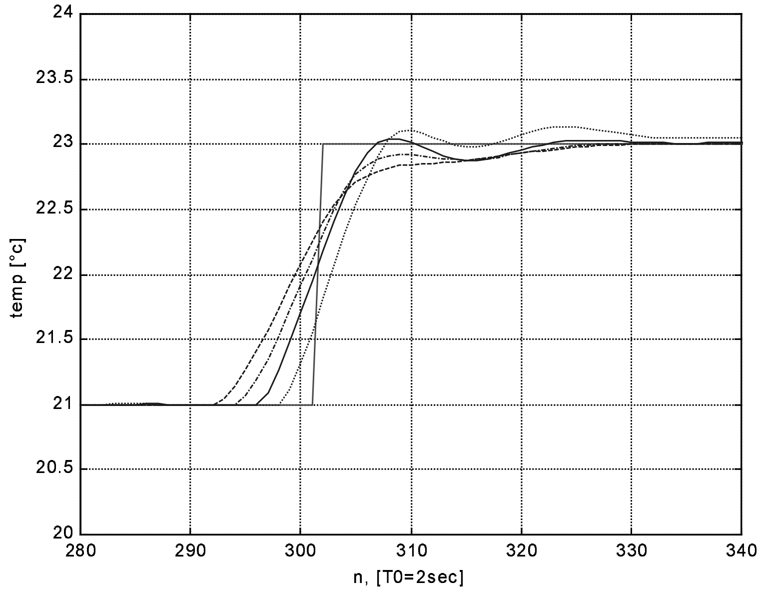

Figure 5. The effect of the prediction horizon

(- - - *p=4*, — *p=6*, - .- *p=8*, -- -- -- *p=10*)

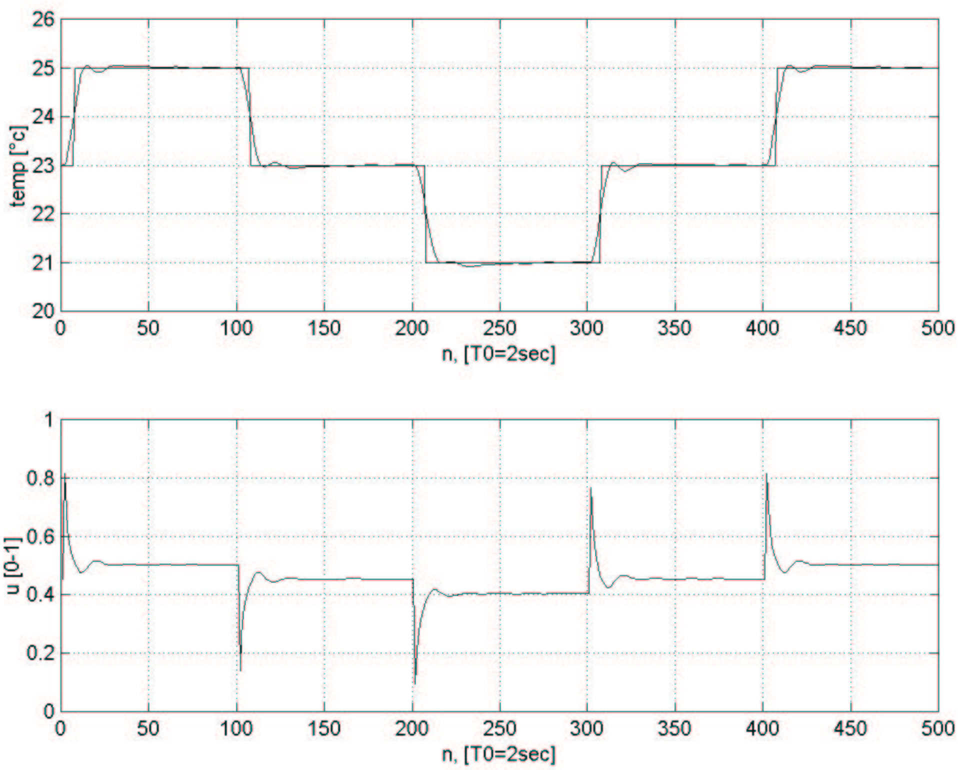

Figure 6. The performance of the PCC controller

(p = 6, **ISE=19.83**)

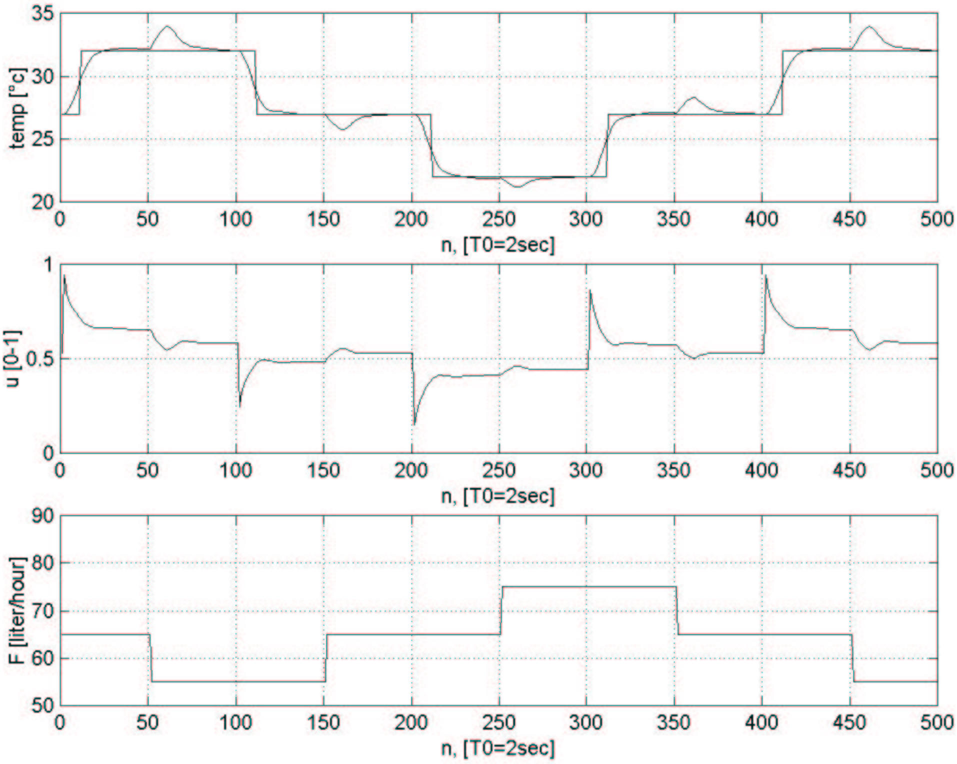

Figure 7. The performance of the controller

at wide range set-point changes and load disturbances