HYBRID FUZZY CONVOLUTION MODEL AND ITS APPLICATION IN PREDICTIVE CONTROL

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ABSTRACT

In this paper a new method for synthesising nonlinear, control-oriented process models is presented. The proposed hybrid fuzzy convolution model (HFCM) consists of a steady-state fuzzy model and a gain-independent impulse response model. The proposed HFCM is applied in model based predictive control of a laboratory-scale electrical water-heater. Simulation and real-time studies confirm that the method is capable of controlling this delayed and distributed parameter system with a strong nonlinear feature.

Keywords: model-based control, predictive control, fuzzy modelling, impulse response model

INTRODUCTION

In recent years a wide variety of model based controller design techniques have been developed. As most of these techniques require models of restricted complexity, first-principles models can not be easily utilised in them. Empirical modelling for control is a possible alternative that allows much greater influence in the model structure. However, empirical modelling based only on input-output data sometimes results in models that have poor prediction performance. Hence, in the area of modelling and identification there is a tendency to blend information of different nature – e.g. experience of operators and designers, measurements, and mathematical equations based on a priori knowledge [1].

As fuzzy modelling and identification is a useful tool for combining knowledge obtained from linguistic rules and information gained from measurement data [2], recent years have witnessed a rapid growth in the use of fuzzy techniques in process modelling. Unfortunately, because of the exponentially increasing memory and information requirements, the construction of fuzzy models of high order systems is a very difficult task. Hence, the use of fuzzy models on high dimensional problems is problematical and requires special identification procedure [3, 4]. An another way to attack this problem is the use of a model structure that is based on local linear model network approach [5], where the model realizes a smooth interpolation between linear models.

The aim of this paper is to develop a new control relevant modelling technique that is based on an another approach, namely block-oriented modelling. The proposed modelling framework assumes that the process can be adequately modelled as a combination of a static nonlinearity (nonlinear steady-state gain) and a gain-independent dynamic part. In recent years it has been shown that such block-oriented model structures are able to tackle nonlinear effects encountered in most chemical processes such as distillation, and pH neutralisation as well as furnaces and reactors [6, 7, 8]. Moreover, these models have been shown to be suitable for utilising different type of information. In these studies it is assumed that the steady-state behaviour of the process is known *a priori* [9, 10]. The disadvantage of this approach is that – as a process gets more complex in its physical description – the first-principles steady-state model tends to become increasingly complicated and computationally intensive, often requiring nonlinear-equation-solving techniques. Moreover, the parameters of the first-principles steady-state model are often laborious to obtain.

In order to solve this problem, the steady-state behaviour of the system is represented by a fuzzy model that is computationally simpler than a first-principles model. The structure (the rules) of the fuzzy model is generated based on linguistic and mechanistic knowledge about the steady-state behaviour of the system, while the parameters of the rule consequences are identified with the help input-output data gathered from the process.

The dynamic part of the block-oriented model is represented by an impulse response model (IRM). In practice, the identification of the parameters of the IRM may be troublesome due to the large number of them [11]. In some cases this problem can be simplified, if the modeller has *a priori* knowledge about the dynamic behaviour of the system. This is especially true for some chemical processes, when the impulse response model relates to the density function of the residence time distribution of the operating unit [12].

The proposed HFCM is applied in model predictive control (MPC) that has been received well in process industry [13]. The implementation of nonlinear models within the MPC algorithm is one of the most challenging tasks of our days.

Fuzzy models have also been incorporated into the MPC scheme. Most of these MPC strategies are based on linear models which linearise the fuzzy process model locally [14, 15]. Other approaches transform the fuzzy model into a step or impulse response model that is valid around the current operating point [16, 17]. The generated control algorithm is an intermediate between these approaches, because for the computationally effective implementation, the effect of the future control outputs are calculated based on the linearized model, while the effect of the previous control signals is estimated by using the nonlinear model itself. Hence, the proposed HFCM can be applied in MPC in a very straightforward way.

The paper is organised as follows. After the description of the hybrid fuzzy convolution model the paper deals with the model-based controller structure. The next part presents an application example, where the control of a laboratory-sized heating system is chosen as a case study. The last section offers the relevant conclusions.

THE HYBRID FUZZY CONVOLUTION PROCESS MODEL (HFCM)

The proposed block-oriented hybrid fuzzy convolution model can be considered as a gain-scheduled convolution model. The output of the model can be formulated as:

$$y_m(k+1) = \underbrace{y_s + K(u_s, x_2, ..., x_n)}_{\substack{steady-state \\ part}} \cdot \underbrace{\sum_{i=1}^{N} g_i(x_2, ..., x_n) \cdot (u(k-i+1) - u_s)}_{\substack{dynamic \\ part}} \tag{1}$$

where the convolution has to handle the gain-independent impulse response model, $g_i(x_2,...,x_n)$, the previous input values, $u(k-i+1)$, over the N model horizon, where u_s denotes the steady-state input and $x_2,...,x_n$ are other operating parameters having effects on the steady-state output $y_s = f(u_s, x_2, ..., x_n)$.

The convolution is multiplied by the steady-state gain

$$K = \frac{\partial f(u_s, x_2, ..., x_n)}{\partial u_s}. \tag{2}$$

According to the choice of the reference (steady-state) point – y_s or u_s – the convolution model can be applied in several way. For instance, if the reference point is chosen as $u_s = \sum_{l=1}^{N} g_l(x_2,...,x_n)\cdot u(k-l+1)$, the proposed model is a Wiener model: $y_m(k+1) = f\left(\sum_{i=1}^{N} g_i(x_2,...,x_n)\cdot u(k-i+1), x_2,...,x_n\right)$. This can be seen as a *parallel model* [18] application of the hybrid convolution model. If the steady-state nonlinearity is invertable, $u_s = f^{-1}(y_s, x_2,...,x_n)$, it is possible to apply the model in *series-parallel model* [18] by choosing the reference as $y_s = y(k)$, where $y(k)$ is the current measured process output.

THE STEADY-STATE PART: THE FUZZY MODEL

The steady-state behaviour of the system is described by zero-order Takagi-Sugeno [19] fuzzy model formulated with a set of rules as follows:

$$r_{i_1,...,i_n}: \text{ if } x_1 \text{ is } A_{1,i_1} \text{ and ... and } x_n \text{ is } A_{n,i_n} \text{ then } y_s = d_{i_1,...,i_n} \quad (3)$$

where $r_{i_1,...,i_n}$ denotes index of the fuzzy rule, *n* is the number of inputs, and $\mathbf{x} = [x_1,..,x_n]^T$ is a vector containing the inputs of the model. $A_{j,i_j}(x_j)$ is the $i_j = 1,2,...,M_j$ -th antecedent fuzzy set referring to the *j*-th input, where M_j is the number of the fuzzy sets on the j -th input domain. Let the first element of the input vector be the steady-state input, $x_1 = u_s$. At a given input, $\mathbf{x}$, the final output of the fuzzy model is inferred by taking the weighted average of the rule consequences, $d_{i_1,...,i_n}$:

$$y_s = \frac{\sum_{i_1=1}^{M_1}\cdots\sum_{i_n=1}^{M_n}\beta_{i_1,..,i_n} d_{i_1,..,i_n}}{\sum_{i_1=1}^{M_1}\cdots\sum_{i_n=1}^{M_n}\beta_{i_1,..,i_n}} \quad (4)$$

where the weights, $0 \le \beta_{i_1,..,i_N} \le 1$, implies the overall truth value of the $i_1,..,i_n$ -th rule calculated as:

$$\beta_{i_1,\ldots,i_n} = \prod_{j=1}^{n} A_{j,i_j}\left(x_j\right). \tag{5}$$

Triangular membership functions were employed for each fuzzy linguistic value as shown in Figure 1, where a_{j,i_j} denotes the cores of fuzzy set A_{j,i_j} :

$$a_{j,i_j} = core(A_{j,i_j}) = \left\{ \ x_j \middle| A_{j,i_j}\left(x_j\right) = 1 \ \right\} \tag{6}$$

Thus, the membership functions are defined as follows:

$$\begin{aligned} A_{j,i_j}\left(x_j\right) &= \frac{x_j - a_{j,i_j-1}}{a_{j,i_j} - a_{j,i_j-1}}, \quad a_{j,i_j-1} \le x_j < a_{j,i_j} \\ A_{j,i_j}\left(x_j\right) &= \frac{a_{j,i_j+1} - x_j}{a_{j,i_j+1} - a_{j,i_j}}, \quad a_{j,i_j} \le x_j < a_{j,i_j+1} \end{aligned} \tag{7}$$

More details on this type of fuzzy model can be found in [20].

At given observations, $x_j \in \left(a_{j,m_j}, a_{j,m_j+1}\right]$, the gain of the steady-state fuzzy model can be computed based on calculating the partial derivative [21]:

$$\begin{aligned} K &= \frac{\partial y_s}{\partial u_s} = \sum_{i_1=m_1}^{m_1+1} \cdots \sum_{i_n=m_n}^{m_n+1} \left[\left(\frac{\Gamma_{i_1-1}(u_s)}{a_{1,i_1} - a_{1,i_1-1}} - \frac{\Gamma_{i_1}(u_s)}{a_{1,i_1+1} - a_{1,i_1}} \right) \cdot \prod_{j=2}^{n} A_{j,j_l}\left(x_j\right) \cdot d_{i_1,\ldots,i_n} \right] \\ \Gamma_{i_1} &= \begin{cases} 1, & if \ u_s \in \left(a_{1,i_1}, a_{1,i_1+1}\right] \\ 0, & if \ u_s \notin \left(a_{1,i_1}, a_{1,i_1+1}\right] \end{cases} \end{aligned} \tag{8}$$

If the reference point is defined by y_s, the $u_s = f_s^{-1}\left(y_s, x_2, \ldots, x_n\right)$ mapping has to be determined in order to be able to use the previous equation. At this point, the inversion problem of the fuzzy model arises. In this study, the inversion method applied in [20] is used.

The most obvious way to identify the steady-state fuzzy model is based on measured steady-state input-output data pair. If there is no steady-state measurements available, the fuzzy model has to be identified based on dynamic training data. The remainder part of this subsection is intended to describe an identification method where the fuzzy model is identified based on this transient data and a previously identified IRM model.

By using a given training data set that has N_t entries, the parameters of the fuzzy model are obtained by minimising the mean square error (MSE) criterion $MSE = \frac{1}{N_t}(\mathbf{y} - \mathbf{W} \cdot \mathbf{d})^T (\mathbf{y} - \mathbf{W} \cdot \mathbf{d})$, where $\mathbf{y} = [y(k+1)^1, \cdots, y(k+1)^{N_t}]^T$ is the vector of the measured outputs and $\mathbf{d} = \left[d_{1,\ldots,1} \ \cdots \ d_{M_1,\ldots,M_n} \right]^T$ is the vector of the rule consequences to be determined. If the HFCM is identified as a Wiener model, the elements of the $\mathbf{W} = \begin{bmatrix} w^1_{1,\ldots,1} & \cdots & w^1_{M_1,\ldots,M_n} \\ \vdots & \ddots & \vdots \\ w^{N_t}_{1,\ldots,1} & \cdots & w^{N_t}_{M_1,\ldots,M_n} \end{bmatrix}$ matrix are $w^t_{i_1,\ldots,i_n} = \prod_{j=1}^{n} A_{j,i_j}\left(x^t_j\right)$. As the best solution of $\mathbf{d}$ is that minimises the MSE criteria, the identification of the rule consequent parameters is a standard linear least-squares estimation problem, $\mathbf{d} = \left(\mathbf{W}^T \mathbf{W}\right)^{-1} \mathbf{W}^T \mathbf{y}$.

THE DYNAMIC PART (THE IMPULSE RESPONSE MODEL)

The identification of the dynamic part of a block-oriented model is a challenging task. In practice, the identification of the parameters of the IRM may be troublesome due to the large number of them [11]. In some cases this problem can be simplified, if the modeller has *a priori* knowledge about the dynamic behaviour of the system. This is especially true for some chemical processes, where the impulse response model relates to the density function of the resident time distribution of the operating unit [10]. In chemical engineering practice, the residence time of a cascade consisting of continuous perfectly mixed operating units is often used to approximate the behaviour of partially known processes, e.g. a distributed parameter system. The density function of the residence time distribution of this "general" process can be described as:

$$\varphi(t,\tau) = \frac{n_c}{\tau} \cdot \frac{\left(n_c \cdot \frac{t}{\tau}\right)^{n_c - 1}}{(n_c - 1)\ !} \cdot \exp\left(-n_c \cdot \frac{t}{\tau}\right) \tag{9}$$

where n_c is the number of the elements of the cascade and τ is the residence time. The discrete impulse response (IRM) model can be easily calculated from this continuous distribution function:

$$g_i = \frac{\varphi(i \cdot \Delta t)}{\sum_{i=1}^{N} \varphi(i \cdot \Delta t)} \tag{10}$$

where Δt denotes the sampling time, i the ith discrete time-step, and N is the model horizon. By using this approach, the identification of the parameters of the IRM is transformed into the identification of the number of the cascade elements, n_c, and the average resident time, τ. This results in a more parsimonious IRM model description, where the variance of the identification problem is decreased by the decrease of the number of the parameters to be estimated. Moreover, if the process dynamic is dependent on the operating parameters, this effect can be easily parameterised by using (9) (10) and a function that estimates the resident time $\tau = f(\mathbf{x})$. An example for the identification of such a function will be presented in application study of the paper.

THE HYBRID FUZZY MODEL BASED PREDICTIVE CONTROLLER

The nonlinear hybrid fuzzy convolution model can be easily applied in model predictive control scheme. The control algorithm is based on the natural division of the system response into *free* and *forced* response terms [22]:

$$y_m(k+t) = y_{forced}(k+t) + y_{free}(k+t) \tag{11}$$

where the forced output, $y_{forced}(k+t)$, depends only on the future inputs,

$$y_{forced}(k+t) = K \sum_{i=1}^{t} s_i \Delta u(k+t-i) \tag{12}$$

where $\{s_i\}$ are the gain independent step response coefficients defined by $s_i = \sum_{j=1}^{i} g_j$; and $\Delta u(k+t-i)$ denotes the change on the control variable: $\Delta u(k+t-i) = u(k+t-i) - u(k+t-i-1)$.

As the previous equation suggests, the forced response is calculated by using a linear model, because the steady-state gain, K, is calculated at the kth time step, and is assumed to be constant during the prediction. In control engineering practice such one step linearization is commonly used for simplifying the highly computational-demanding optimisation task [14, 15]. The proposed method differs from these approaches in the calculation of the *free response* of the system that represents the effect of the previous control signals that can interpreted as the future response of the process assuming that the process input is constant during the prediction horizon, H_p. Hence, the nonlinear hybrid fuzzy convolution model is used to generate this *free response*, $y_{free}(k+i) = f_s\left(\left(f_s^{-1}(y(k), x_2,..., x_n) + Q_i\right), x_2,..., x_n\right)$, where the Q_i coefficients are [23]:

$$Q_i = \sum_{t=1}^{i} \sum_{j=t+1}^{N} g_j \Delta u(k+t-j), \qquad i = 1,2,..,H_p . \tag{13}$$

The future incremental control actions, $\Delta\mathbf{u} = [\Delta u(k), \ldots, \Delta u(k+H_c)]^T$, are obtained by minimising the following cost function:

$$\min_{\Delta\mathbf{u}} = \left(\mathbf{r} - \left(K\mathbf{S}\Delta\mathbf{u} + \mathbf{y}_{free}\right)\right)^2 + \lambda\Delta\mathbf{u}^2 \tag{14}$$

where $\mathbf{r} = [r(k+1), \ldots, r(k+H_p)]^T$ denotes the future set-point values, $\mathbf{y}_{free} = [y_{free}(k+1), \ldots, y_{free}(k+H_p)]^T$ the predicted free-response, and $\mathbf{S}$ is the gain independent dynamic matrix:

$$\mathbf{S} = \begin{bmatrix} s_1 & 0 & 0 & \cdots & 0 \\ s_2 & s_1 & 0 & & 0 \\ s_3 & s_2 & s_1 & \ddots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ s_{H_c} & s_{H_c-1} & s_{H_c-2} & & s_1 \\ \vdots & \vdots & \vdots & & \vdots \\ s_{H_p} & s_{H_p-1} & s_{H_p-2} & \cdots & s_{H_p-H_c+1} \end{bmatrix}_{H_p \times H_c} \tag{15}$$

The move suppression coefficient, λ, employs a punishment for the variation of the manipulated variable. For nonlinear processes this constant can be gain-scaled by expressing it as a product of a scaled move-suppression coefficient, γ, and the square of the process gain, $\lambda = \gamma \cdot K^2$ [24].

If the process constraints are not taken into account, the previous minimisation problem can be solved effectively by least-squares method,

$$\Delta \mathbf{u} = \frac{1}{K} \cdot \left(\mathbf{S}^T \cdot \mathbf{S} + \gamma \cdot \mathbf{I} \right)^{-1} \cdot \mathbf{S}^T \cdot \mathbf{e}, \tag{16}$$

where **e** is the vector of the estimated errors $\mathbf{e} = \mathbf{r} - \mathbf{y}_{free}$, and **I** is a unity matrix.

The controller has three parameters. These are the prediction horizon, H_p, the control horizon, H_c, and the gain independent move suppression coefficient, γ. The prediction horizon should roughly be equal to the 60% of the open loop settling time to ensure controller stability. When the process in nonlinear, the open-loop settling time is changing with the operating point. According to this effect, the prediction horizon can be adapted during the operation. A simpler solution is setting the prediction horizon equal to the 60% of maximum of the settling time. In the application study of this paper this later approach is followed. The control horizon should be practical $H_c \in [1, 6]$ [22]. As the weighting of Δu is used to avoid extreme changes in the control signal, the choice of γ in the objective function (16) always depends on the desired control performance.

EXAMPLE:

MODEL PREDICTIVE CONTROL OF AN ELECTRICAL WATER-HEATER

A laboratory-scale electrical water-heater is considered to illustrate the advantages of the proposed modelling and model based control approach.

THE PROCESS EXAMINED

The schematic diagram of the water-heater is depicted in Figure 2. The water comes from the water pipeline into the heater through a control valve, (CV). After measuring its flow rate, *F*, the water passes through a pair of metal pipes containing a cartridge heater. The control task is to maintain the outlet temperature of the water, T_{out}, on desired value at different flow rates, by adjusting the heating control signal, *u*, of the cartridge heater. As the performance of the heating cartridge is a sinusoidal function of the control signal (in voltage), at a given flowrate the process considered as a Hammerstein system, because it has nonlinear steady-state behaviour. Moreover, by changing the flowrate, the process has nonlinear static and dynamic nonlinearity. The detailed description of the experimental setup and its first-principles model can be found in [10]

THE HYBRID FUZZY CONVOLUTION MODEL OF THE HEATING SYSTEM

The steady-state output of the process, $y_s = T_{out}$, depends on the water flow-rate, F, and the steady-state heating signal, u_s. The fuzzy model can easily represent this relationship:

$$r_{i_1,i_2}: \text{ if } F \text{ is } A_{1,i_1} \text{ and } u_s \text{ is } A_{2,i_2} \text{ then } y_s = d_{i_1,i_2} \tag{17}$$

For good modelling performance 4 and 7 antecedent fuzzy sets were utilised on the input universes (F, u_s). At the design of the fuzzy model heuristic knowledge was used that is based on our empirical knowledge about the process steady-state behaviour. Hence, more fuzzy sets were defined on the operating regions where the steady-state behaviour of the process is changing. The resulted fuzzy sets are depicted in Figure 3 and 4. The fuzzy model was identified by using least-squares error method and steady-state and transient input-output data. In the simulation example together 250 training data samples were used. Figure 5. shows the surface representation of the resulted steady-state fuzzy model. As this figure suggests, the process has highly nonlinear steady-state characteristic.

When examining the shape of the impulse response curves generated from the process, it has turned out that they are formally analogous with the density functions of the residence time distribution of a cascade consisting of CSTRs. The fitting of equations (9, 10) to the measured impulse-response curves at different flow-rates leads to the choice of $n_c = 2$ on the whole operating interval. Thus, g_i, which is in fact the normalised density function of the residence time distribution, can be expressed as:

$$g_i = \frac{q^2 \cdot i \cdot \Delta t \cdot \exp(-q \cdot i \cdot \Delta t)}{\sum_{i=1}^{N} q^2 \cdot i \cdot \Delta t \cdot \exp(-q \cdot i \cdot \Delta t)} . \tag{18}$$

where Δt =2 sec denotes the sampling time, i the ith discrete time-step, and N is the number of the time steps (model horizon). As the process dynamic (the resident time) is changing with the flowrate, q is a function of the flow-rate, F. The relation q-F is properly approximated by an equation fitted to the measured impulse response :

$$q = 0.064 \cdot \left(\frac{F}{150}\right)^{0.11}. \tag{19}$$

This resulted in a good approximation of the measured IRM (Figure 6). By using this approach the complete process dynamic is identified with three parameters and *a priori* knowledge about the process dynamic. This example is really shows the advantage of the proposed method over conventional techniques where different local linear models (in these case different IRMs) are identified in different operating regions, and the global nonlinear model is defined as an interpolation between these linear models resulting in an overparameterised model.

THE HYBRID FUZZY CONVOLUTION MODEL BASED CONTROL OF THE HEATING SYSTEM

The HFCM based controller is implemented to the heating process through the following algorithmic steps:

1. calculation of q with (19) by making use of the value of the measured flow-rate (F),
2. calculation of the impulse response model from (18),
3. calculation of u_s from $y_s = y(k)$, considering the inversion of the fuzzy model,
4. calculation of the value of the steady-state gain by (8).
5. calculation of **S** and **e**
6. calculation of the controller output from the first element of the calculated $\Delta\mathbf{u}$ vector generated from (16)

In order to compare the achieved control performances, the following performance index is used:

$$\text{Perf} = ISE + \chi \cdot ISdU = \sum_{k=1}^{I} e(k)^2 + \sum_{k=1}^{I} \chi \cdot \Delta u(k)^2 \tag{22}$$

where $e(k) = r(k) - y(k)$ is the controller error at the *k*th discrete time step, and $I = \frac{t_{\max}}{\Delta t}$ is the maximal number of time steps. This error criterion employs a punishment member for the variation of the manipulated variable. The value of $\chi = 1000$ is chosen according to the value of $\gamma \cdot K^2$, where $K \in [0 - 60]$ and the gain independent move suppression coefficient γ is around 0 to 20.

In order to show the advantages and disadvantages of the proposed method, a simulation study has been performed where the controller is compared to an optimised PID and a gain-scheduled dynamic matrix controller [10]. This study is performed based on the simulation model of the process described in [10].

In spite of the fact that PID controllers meet hard difficulties when employed in control of nonlinear systems, they are frequently used in process industries. Therefore, the PID control solution is considered as a first evaluation of the control task. The parameters of the PID controller were determined by minimising the cost function (22) with Sequential Quadratic Programming method. The control performance of the optimised PID controller is shown in Figure 7. As Figure 8 shows, the proposed HFCM based controller provides superior control performance in this operating range (*Perf=29.4* for the HFCM based MPC, and *Perf=114.7* for the PID controller).

In order to make a more sophisticated comparison, the presented approach is compared to a gain-scheduled dynamic matrix controller (DMC). This controller also utilises a gain-scheduled convolution model, but the steady-state part of the model is based on a first-principles model. For handling the first-principles steady-state model, an iterative Newton algorithm is utilised for the calculation of the steady-state relationship. In this comparison study not only the set-point, but also the water flow-rate is varied. This makes the control task really challenging. As Figure 9 shows, by using the proposed HFCM based controller, the achieved control performance is similar to the performance of the first-principles model based algorithm [10]. This similar control performance suggests that the advantage of the proposed approach lies in the facility of using smaller amount of *a priori* knowledge for modelling, and less computational demand to implement a control oriented process model.

The presented modelling and control approach has been applied to the real process. For real-time experiments the fuzzy model was identified based on input-output steady-state and dynamic data obtained from the real process. The achieved control performances correspond to the previously presented simulation results. Therefore, only the effect of the tuning parameters γ, H_c, H_p on the real-time control performance is discussed. As Table 1 shows, the effect of these parameters corresponds to that observable in the case of other predictive controllers [24], because the output response becomes less oscillatory and more sluggish as the prediction horizon, H_p, and the gain independent move suppression coefficient, γ, are increased; and at given H_p and γ the increase of the control horizon, H_c, has an opposite effect.

The performance of the best two controllers in real-time application is shown in Figure 10 and 11. As these figures show, the resulted control performances are worse than the performances obtained on the simulation experiments. This performance degradation is caused by the unmeasured process disturbances like the noise of the temperature sensor. However, the achieved control performance is superior to the real-time results obtained by PID and linear DMC applications presented in [10].

CONCLUSIONS

This paper presented the capabilities of hybrid fuzzy convolution models in modelling and model predictive control for a delayed and distributed parameter system with a nonlinear feature. The proposed hybrid fuzzy convolution model (HFCM) consists of a steady-state fuzzy model and a gain-independent impulse response model. The model is applied in model predictive control.

Some people attack the fuzzy control community by stating that the final control and/or modelling algorithm just boils down to a gain-scheduling which could actually be obtained by other interpolation methods, too. This is especially valid for the presented block-oriented fuzzy modelling approach, because block-oriented models generalise the well-known gain-scheduling concept of nonlinear control [25]. However, fuzzy techniques provide a man–machine interface, which facilitates the acceptance, validation and transparency of the process model very much. The question is not whether the tool boils down to a simple algorithm: it is very convenient from the point of view of computational effort and realisation [26]. The presented nonlinear hybrid fuzzy convolution model uses this advantage of fuzzy systems in the representation of the steady-state behaviour of the system.

The other advantage of the presented proposed modelling approach is that it tries to combine knowledge about the system in form of *a priori* knowledge and measured data in the identification of a control relevant model.

The control of a laboratory-sized heating system has been chosen as a realistic nonlinear case study for the demonstration of the applicability of the proposed method. It has been shown that the proposed model can be implemented in a model predictive controller in a straightforward way.

Simulation example and real-time results illustrate the potential offered by the hybrid fuzzy convolution model based predictive controller. The results support the assumption that the advantage of the proposed approach comes from the combination of a priori, linguistic and input-output data based information. By using different sources of information, smaller amount of *a priori* knowledge is required to achieve similar control performance compared with first-principles model based control solutions. This is coupled with a further advantage, that less computational effort is needed to implement the hybrid fuzzy convolution control-oriented process model for predictive control compared to other pure *a priori* or pure fuzzy model based techniques.

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NOMENCLATURE

The convolution model

g_i	parameters of the impulse response model
N	model horizon
K	steady-state gain
u_s	steady-state input
y_s	steady-state output
$y(k)$	current measured process output
$\varphi(t)$	density function of the residence time distribution
n_c	number of the elements of the cascade
τ	average residence time
$r_{i_1,\ldots,i_n}$	index of the fuzzy rule
n	number of the inputs of the fuzzy model
$\boldsymbol{x} = [x_1,..,x_n]^T$	n vector containing all inputs of the fuzzy model
M_j	number of the fuzzy sets on the j -th input domain
$A_{j,i_j}(x_j)$	$i_j = 1,2,\ldots,M_j$ -th antecedent fuzzy set referring to the j-th input
a_{j,i_j}	cores of the A_{j,i_j} fuzzy set
$d_{i_1,\ldots,i_n}$	consequence parameter of the $i_1,\ldots,i_n$ -th rule
$\beta_{i_1,\ldots,i_n}$	weights of the $i_1,\ldots,i_n$ -th rule

The controller

$y_{forced}(k+t)$	predicted forced response
$y_{free}(k+i)$	predicted free response
$\{s_i\}$	gain independent step response coefficients
H_c	control horizon
H_p	prediction horizon
$\Delta\mathbf{u} = [\Delta u(k),\ldots,\Delta u(k+H_c)]^T$	future incremental control actions

$\mathbf{r} = \left[r(k+1), \ldots, r(k+H_p)\right]^T$	future set-point values
$\mathbf{y}_{free} = \left[y_{free}(k+1), \ldots, y_{free}(k+H_p)\right]^T$	predicted free-response
$\mathbf{S}$	gain independent dynamic matrix
λ	move suppression coefficient
γ	scaled move-suppression coefficient
$\mathbf{e}$	vector of estimated errors over the prediction horizon

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Table 1. Real-time control results, the effect of , γ , H_c , H_p , tuning parameters

H_c	H_p	γ	*ISE*	$\chi \cdot ISdU$	*Perf.*
1	*10*	*0*	782.9	418.2	1202.1
	15	0	464	48.7	512.7
	20	*0*	642.9	18.3	661.2
2	*15*	*5*	730	43	773
		10	651	17	668
		20	796.7	8.5	805.2
	20	*5*	577	14	591
		10	*573*	*10.8*	*583.8*
		20	645.2	9.1	654.3

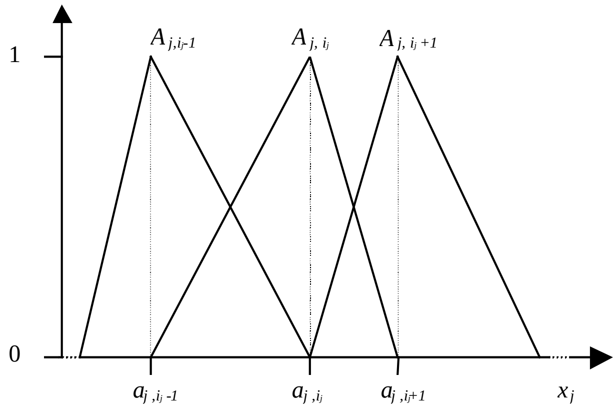

Figure 1 Membership functions used for the fuzzy model

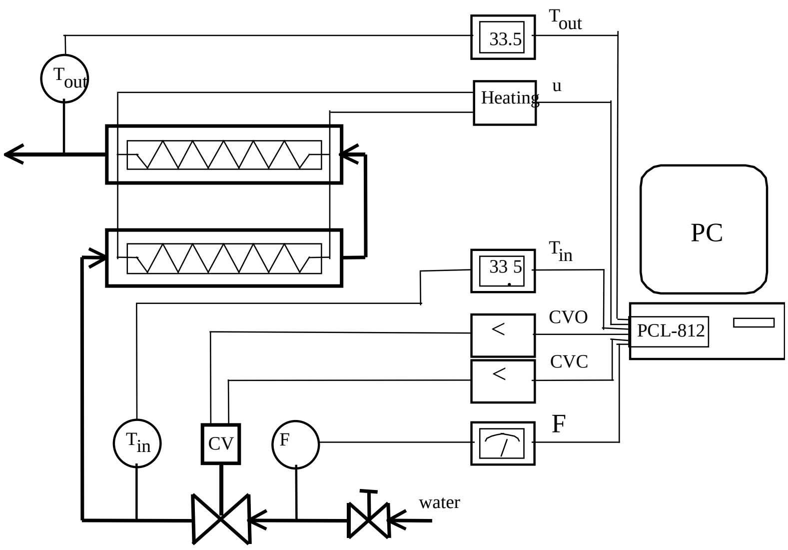

Figure 2 The scheme of the physical system

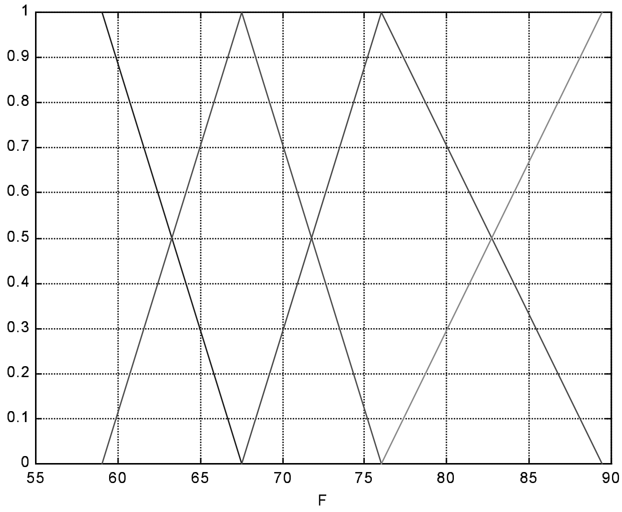

Figure 3 Membership functions on the input flow rate, *F*

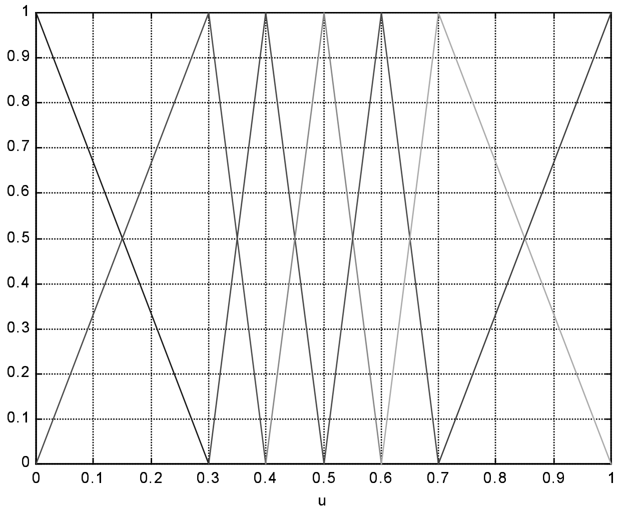

Figure 4 Membership functions on the control signal, u_s

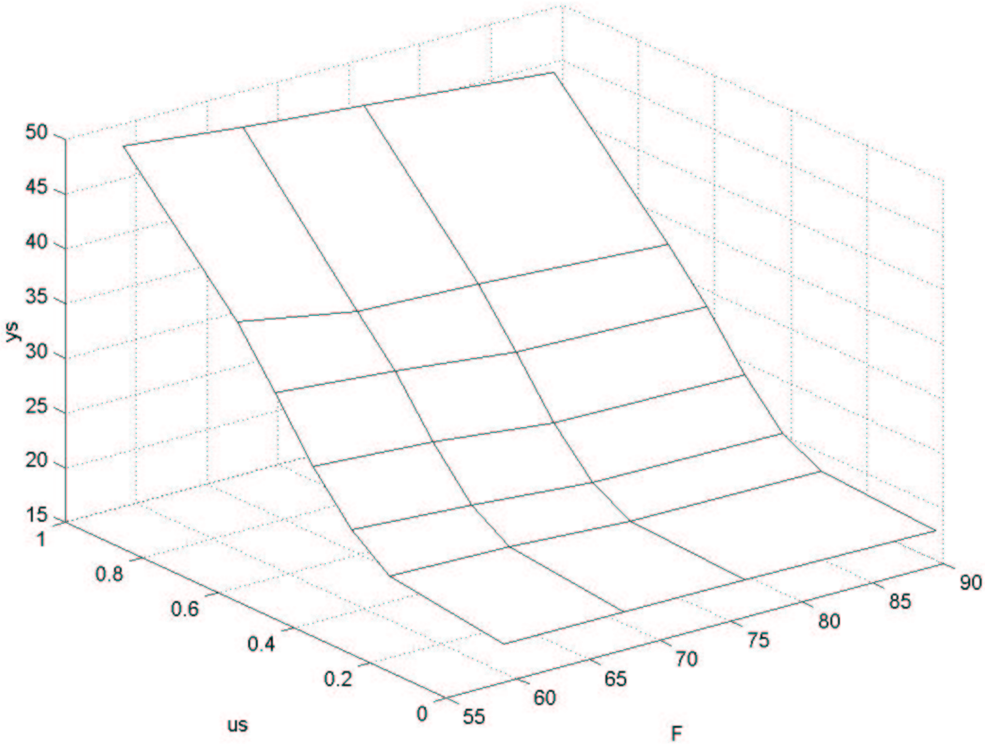

Figure 5 The surface representation of the fuzzy model of the steady-state behaviour

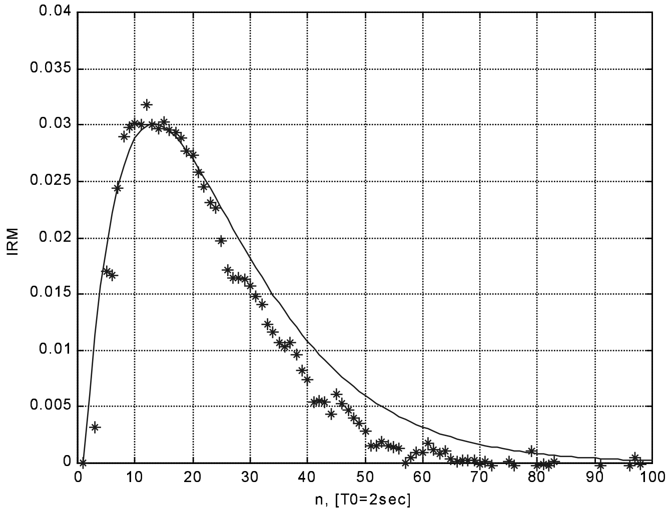

Figure 6 Measured and fitted impulse response model at given flow rate (* measured, - fitted)

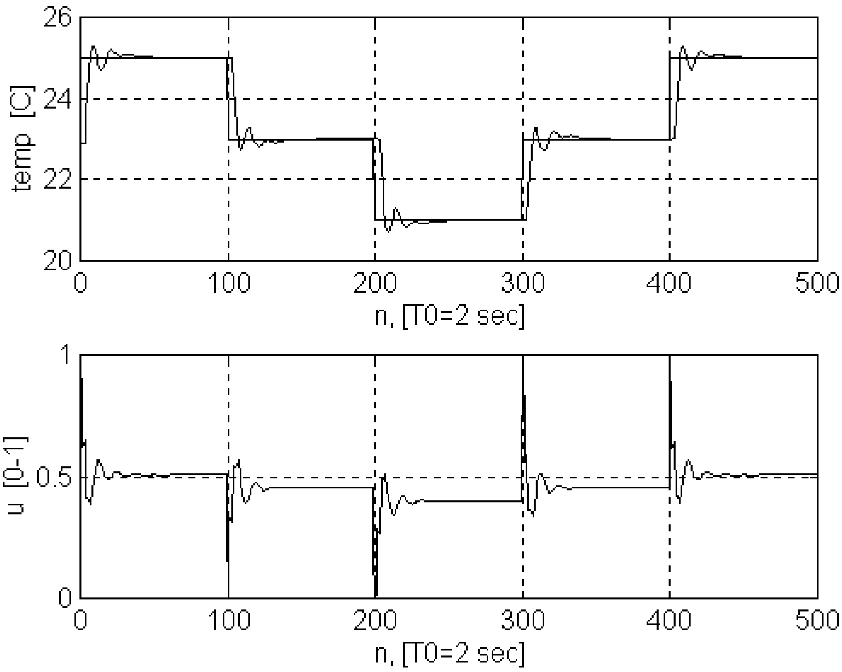

Figure 7. The performance of the optimised PID controller (simulation, Perf=114.7)

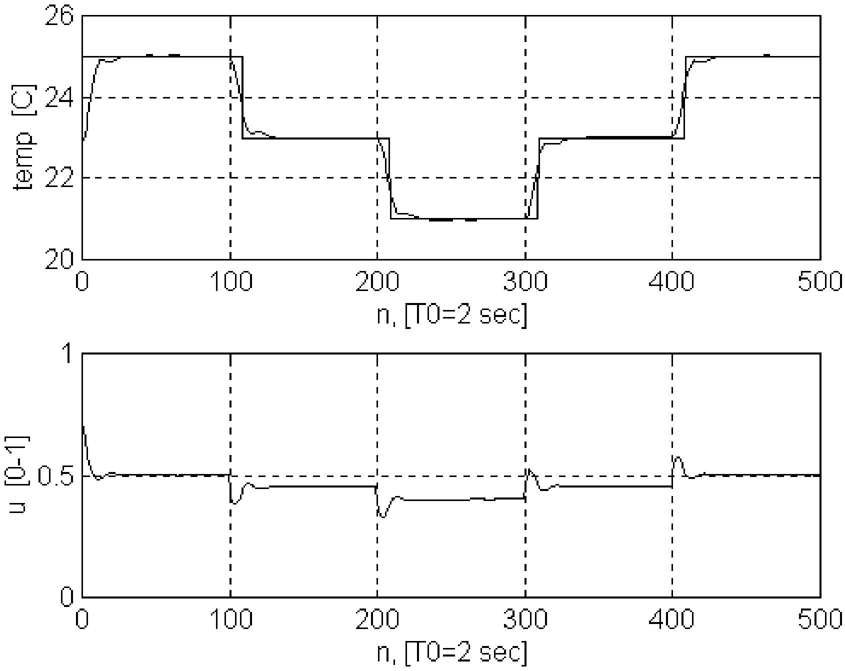

Figure 8. The performance of the HFCM based controller

(simulation, $\gamma = 4 \cdot 10^{-3}$, $H_c = 1$ $H_p = 10$, Perf=29.41)

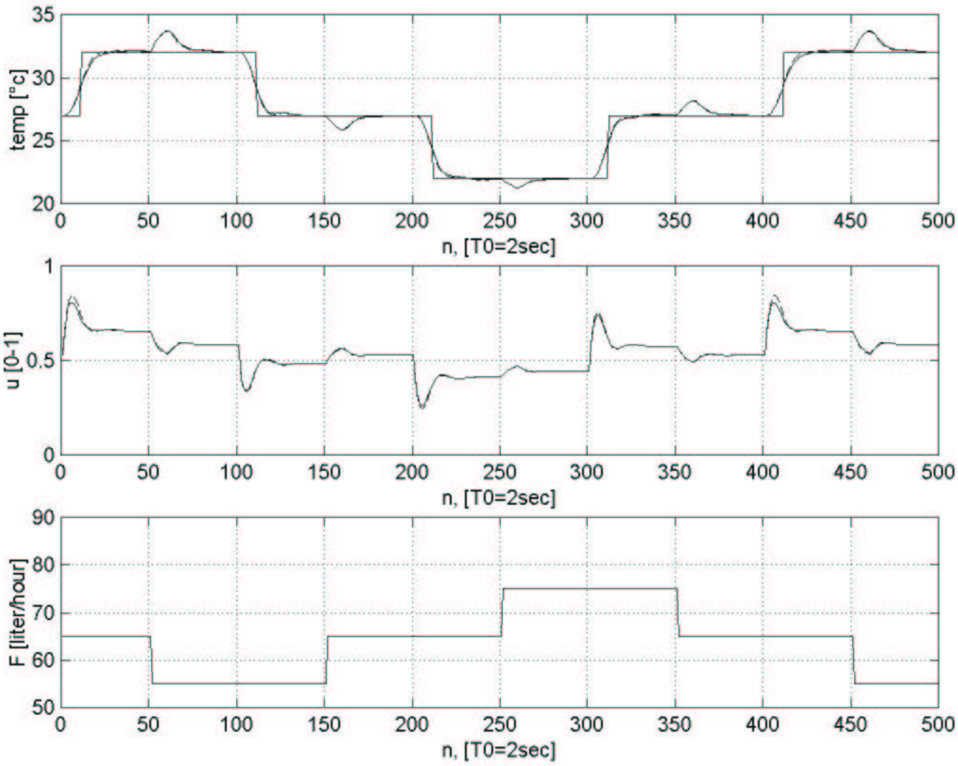

Figure 9 Comparison of the hybrid fuzzy model based controller, '–'

and an *a priori* model based gain-scheduled DMC controller, '--'

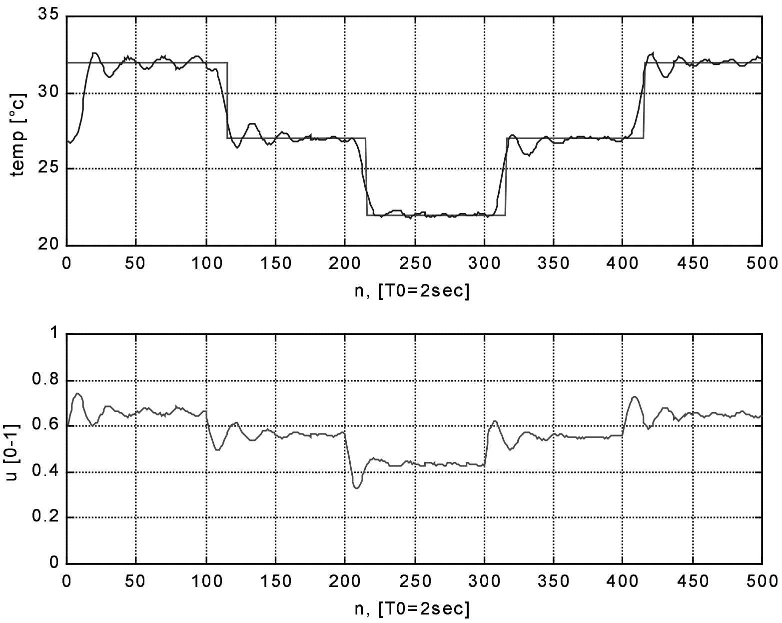

Figure 10 Real time control performance by using the HFCM based controller

(γ =0, H_c =1, H_p =15)

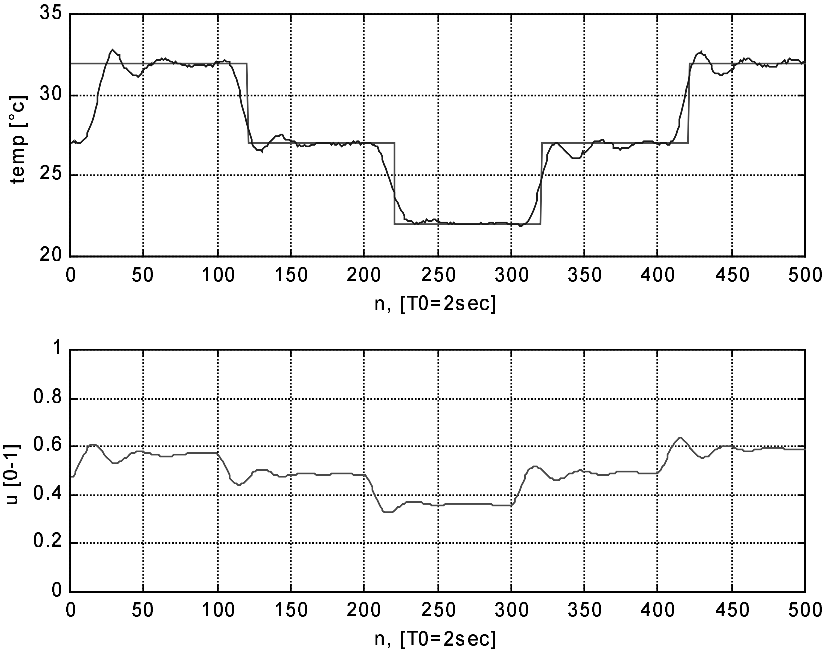

Figure 11 Real time control performance by using the HFCM based controller

(γ=10, H_c=2, H_p=20)