Local and Global Identification for Fuzzy Model Based Control

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ABSTRACT

There are two approaches to extract a linear model from a Takagi-Sugeno fuzzy model for model based control. The first local approach obtains the linear model by interpolating the parameters of the local models in the TS model, while the second one is based on linearization by Taylor expansion. The locally interpreted interpolated model is not identical to the model obtained by the linearization of the fuzzy model. The paper analyzes the origin of this difference with regard to the applied identification method and the application of the resulted model in model predictive control. In order to keep the analysis simple and transparent, a fuzzy model of a Hammerstein system is studied.

1 INTRODUCTION

Fuzzy identification is an effective tool for the approximation of uncertain nonlinear dynamic systems on the basis of measured input-output data [2]. The Takagi-Sugeno (TS) fuzzy model [7] is often used to represent nonlinear dynamic systems, by interpolating between local linear, time-invariant (LTI) ARX models. These control-relevant models can be effectively used in model predictive control (MPC) [4]. To avoid non-convex optimization in the MPC problem, instead of a nonlinear fuzzy model, local linear models extracted from the TS model are used [6] by the MPC algorithm.

There are two approaches to extract a linear model from the fuzzy model around a given operating point. The first approach is based on the fact that the TS model interpolates between local linear models. Hence, the extracted linear model is obtained by interpolating the parameters of the local models in the TS model. The second approach extracts the parameters of the linear model by Taylor expansion. The locally interpreted interpolated model is not identical to the model obtained by this Taylor expansion based linearization of the fuzzy model [1].

For the identification of a fuzzy model, also two approaches can be followed. With the global approach the parameters of all rule consequences are estimated within one identification problem, yielding an optimal predictor. The local parameter estimation does not estimate all parameters simultaneously. It rather divides this task into a set of weighted least-squares problems [2].

As will be shown in the paper, only consequent parameters obtained by local estimation (weighted least squares) can be interpreted locally and used as a Linear Parameter Varying (LPV) model in the MPC where the extracted model parameters are interpolated. Parameters obtained by global identification do not lend themselves to the LPV interpretation. Such a model is only applicable where the extracted models are obtained by Taylor-series linearization.

The paper is organized as follows. In Section 2, the applied TS fuzzy model of dynamic systems is introduced. Section 3 addresses the control relevant extraction of the linear models from this model. Section 4 studies the effect of the applied identification method through an example about modeling and model based control of a Hammerstein system with a polynomial nonlinearity. Conclusions are given in Section 5.

2 Takagi-Sugeno fuzzy model of a dynamic system

This paper presents a study of steady-state behaviour of nonlinear dynamic system represented by a fuzzy model as a Nonlinear AutoRegressive with eXogenous input (NARX) model. This model establishes a nonlinear relation between the past inputs and outputs and the predicted output:

$$y(k+1) = F\big(y(k), \ldots, y(k-n_y+1), u(k-n_d), \ldots, u(k-n_u-n_d+1)\big)\,. \tag{1}$$

Here, n_y and n_u are the maximum lags considered for the output and input terms, respectively, n_d is the discrete dead time, and F represents the mapping of the fuzzy model.

The Takagi-Sugeno (TS) fuzzy model [7] of a nonlinear dynamic system interpolates between local linear, time-invariant (LTI) ARX models,

$$\begin{aligned} R_j: &\quad \texttt{If } z_1(k) \texttt{ is } A_{j,1} \texttt{ and } \ldots \texttt{ and } z_n(k) \texttt{ is } A_{j,n} \\ \texttt{then} &\quad y^j(k+1) = \sum_{i=1}^{n_y} a_{i,j} y(k-i+1) + \sum_{i=1}^{n_u} b_i^j u(k-i-n_d+1) + c_j\,, \end{aligned} \tag{2}$$

where the elements of $\mathbf{z}(k)$ "scheduling vector" are usually a subset of $\{y(k), \ldots, y(k-n_y+1), u(k-n_d), \ldots, u(k-n_u-n_d+1)\}$, $A_{j,i}(z_i)$ is an antecedent fuzzy set for the $i = 1 \ldots n$ th input in the jth rule. The same symbol is used to denote a fuzzy set and its membership function. For a given input, the output of the fuzzy model, $y(k+1)$, is inferred by computing the weighted sum of the rule consequences

$$y(k+1) = \sum_{j=1}^{n_j} \beta_j y^j(k+1), \tag{3}$$

where the weight, $0 \le \beta_j \le 1$, is the normalized overall truth value of the jth rule calculated as

$$\beta_j(\boldsymbol{z}) = \frac{\prod_{i=1}^{n} A_{j,i}(z_i)}{\sum_{k=1}^{n_j} \prod_{i=1}^{n} A_{k,i}(z_i)}\,. \tag{4}$$

3 EXTRACTION OF LINEAR MODELS FOR MPC

Model-based predictive control (MPC) is a powerful method to control constrained and multivariable systems [3]. A problem for the real-time control of nonlinear systems is that a nonlinear (and usually non-convex) optimization problem must be solved at each sampling period by the MPC. To avoid non-convex optimization, a single or a set of local linear models are utilized in the MPC, instead of a single nonlinear plant model [6]:

$$y(k+1) \;=\; \sum_{i=1}^{n_y} a_i(\boldsymbol{z}) y(k-i+1) + \sum_{i=1}^{n_u} b_i(\boldsymbol{z}) u(k-i-n_d+1) + c(\boldsymbol{z})\,, \tag{5}$$

where the parameters of the extracted linear model are function of the operating point of the fuzzy model. As will be presented in the remaining part of this section, two approaches can be used for this extraction purpose.

3.1 Extraction Based on Interpretation

As the TS fuzzy model interpolates between LTI ARX models, it can be regarded as a linear parameter-varying (LPV) system. In this case, the parameters of the extracted linear model are calculated as:

$$\begin{aligned} a_{i,LPV}(\boldsymbol{z}) &= \sum_{j=1}^{n_j} \beta_j(\boldsymbol{z}) a_{i,j},\; i = 1, \ldots, n_y\,, \\ b_{i,LPV}(\boldsymbol{z}) &= \sum_{j=1}^{n_j} \beta_j(\boldsymbol{z}) b_{i,j},\; i = 1, \ldots, n_u\,, \\ c_{LPV}(\boldsymbol{z}) &= \sum_{j=1}^{n_j} \beta_j(\boldsymbol{z}) c_j\,. \end{aligned} \tag{6}$$

3.2 Extraction Based on Global Interpretation

An another interpretation considers the fuzzy model as a nonlinear input-output function mapping. In this case, the linear model is extracted by a Taylor expansion of the function:

$$
\begin{aligned}
a_{i,gl}(\boldsymbol{z}) &= \frac{\partial f(.)}{\partial y(k-i+1)},\ i=1,\dots,n_y\,,\\
b_{i,gl}(\boldsymbol{z}) &= \frac{\partial f(.)}{\partial u(k-i-n_d+1)},\ i=1,\dots,n_u\,,\\
c_{gl}(\boldsymbol{z}) &= f(.)-\left(\sum_{i=1}^{n_y} a_{i,gl}(\boldsymbol{z})y(k-i+1)+\sum_{i=1}^{n_u} b_{i,gl}(\boldsymbol{z})u(k-i-n_d+1)\right)\,.
\end{aligned}
\tag{7}
$$

The globally extracted parameters of the fuzzy model, $a_{i,gl}, b_{i,gl}, c_{gl}$ are not equal to the parameters based on the LPV interpretation, $a_{i,LPV}, b_{i,LPV}, c_{LPV}$, because the previous equations can be written as:

$$
\begin{aligned}
a_{i,gl}(\boldsymbol{z}) &= a_{i,LPV}(\boldsymbol{z})+\sum_{j=1}^{n_j}\frac{\partial \beta_j(\boldsymbol{z})}{\partial y(k-i+1)}a_{i,j},\ i=1,\dots,n_y\,,\\
b_{i,gl}(\boldsymbol{z}) &= b_{i,LPV}(\boldsymbol{z})+\sum_{j=1}^{n_j}\frac{\partial \beta_j(\boldsymbol{z})}{\partial u(k-i-n_d+1)}b_{i,j},\ i=1,\dots,n_u\,.
\end{aligned}
\tag{8}
$$

4 EFFECT OF THE IDENTIFICATION METHOD

4.1 Local and Global Identification

To identify the consequent parameters of the fuzzy model, a global or a local approach can be followed. With the global approach the parameters of the rule consequents are estimated within one identification e.g. in one global least-squares problem, yielding an optimal predictor. Therefore, the generated local models are not necessarily the local linearizations of the nonlinear system. The local identification method forces the local linear models to fit the system separately and locally, resulting rule consequents that are local linearizations of the nonlinear system [2]. The local identification can be represented by n_j weighted least-squares estimation.

4.2 Example: Modeling a Hammerstein System

The analysis of the dynamic behaviour of a fuzzy model that represents a general nonlinear dynamic system is a complex task, because the different dynamic behavior of the local models complicate the model description. In order to simplify this problem, a fuzzy model of a Hammerstein systems is analyzed. This system consists of a series connection of a memoryles nonlinearity and linear dynamics, see Figure 1.

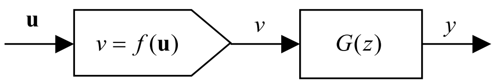


Figure 1. *A series combination of a static nonlinearity and a linear dynamic system.*

The NARX representation of this system is:

$$
y(k+1) = \sum_{i=1}^{n_y} a_i y(k-i+1)+\sum_{i=1}^{n_u} b_i\left\{f_s\left(u(k-i-n_d+1)\right)\right\},
\tag{9}
$$

where $f_s(\cdot)$ is a static (memoryless) nonlinearity. If the gain of the linear subsystem equals to one,

$$
\frac{\sum_{i=1}^{n_u} b_i}{1-\sum_{i=1}^{n_y} a_i}=1,
\tag{10}
$$

the $f_s(\cdot)$ function describes the steady-state behavior of the system. The structure of the fuzzy model used to approximate this type of systems is

$$R_j: \quad \texttt{If}\ u(k)\ \texttt{is}\ A_{j,1}\ \texttt{then}\ y^j(k+1) = \sum_{i=1}^{n_y} a_i y(k-i+1) + \sum_{i=1}^{n_u} b_{i,j} u(k-i-n_d+1) + c_j \,, \tag{11}$$

since (9) is nonlinear in the input $\boldsymbol{z}(k) = u(k)$.

The examined first-order Hammerstein system is $y(k+1) = ay(k) + bf_s\left(u(k)\right)$, where the f_s nonlinearity is represented by a second-order polynomial, $f_s\left(u(k)\right) = 3u(k)^2 - 5u(k) + 6$, and the linear dynamic part has a unity gain, $b = 0.1, a = 0.9$. The studied fuzzy model interpolates between two linear models. The operating region of these local models are represented by two triangular fuzzy sets:

$$A_{1,1}(u(k)) = \frac{t_2 - u(k)}{t_2 - t_1},\ A_{2,1}(u(k)) = \frac{u(k) - t_1}{t_2 - t_1},\ t_1 \leq u(k) < t_2 \,.$$

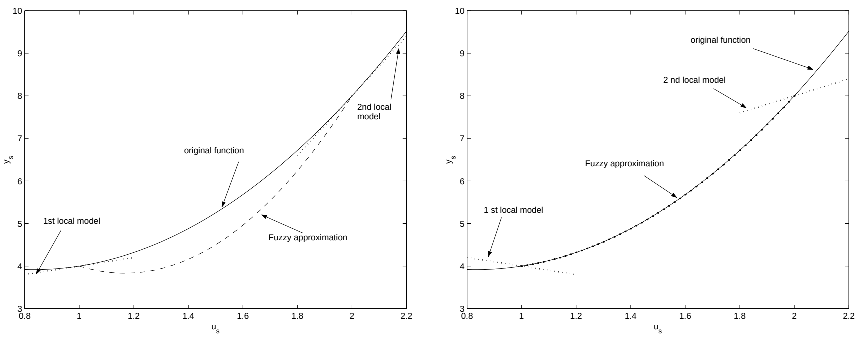
In this paper, instead of the identification based on input-output data, the model is generated directly from the process that is assumed to be known. Because the local identification method forces the local linear models to fit the data locally, the parameters of the locally identified fuzzy model are obtained by the linearization of the fuzzy model at $u(k) = t_1$ and $u(k) = t_2$, $b_{1,LPV} = 0.1, b_{2,LPV} = 0.7, c_{1,LPV} = 0.3, c_{2,LPV} = -0.6$.

The global identification fits the local models globally in one identification problem. Because the studied Hammerstein system is assumed to be known, the parameters of the globally identified fuzzy system can be calculated directly based on the parameters of the Hammerstein system. This task is easy, because the fuzzy model makes a second-order polynomial approximation of the system that has a second-order polynomial nonlinearity. Therefore, the parameters of the globally identified fuzzy model can be calculated from a set of equalities, $b_{1,gl} = -0.1, b_{2,gl} = 0.2, c_{1,gl} = 0.5, c_{2,gl} = 0.4$.

4.3 Comparison of the Steady-state Behaviours

The steady-state behaviour of a model plays crucial role in the performance of the MPC. Hence, in this section, the steady-state behaviour of the identified models are compared.

As Figure 2 shows, the locally identified fuzzy model gives a bad approximation of the steady-state behaviour of the system. Moreover, the fuzzy model represents an input multiplicity, while the original system has a monotonous steady-state behaviour. This effect is caused by the undesirable properties of the weighted mean inference method applied in the TS fuzzy model [2], since the fuzzy model cannot approximate both the value of the function and its derivative. This suggest that the locally identified model should only be used for local LPV interpretation of the system (e.g., for gain scheduling).



(a) Locally identifi ed model. *(b) Globally identifi ed model*

Figure 2. *Steady-state characteristic of the identifi ed fuzzy models.*

As Figure 2 shows, the globally identified model is able to describe the steady-state behaviour of the system and it can be used for global dynamic interpretation, however, the local models do not relate to the linearization of the system. Hence, the LPV interpretation of this model is meaningless.

The effect of the presented differences to the MPC application of the models will be presented in the next subsection.

4.4 Effect on the Control Performance

A generalized predictive controller (GPC) has been designed for the considered process, using the fuzzy models obtained with the proposed identification method. The GPC algorithm computes the control sequence $\{u(k+j)\}$ such that the following quadratic cost function [3] is minimized:

$$J(H_{p1}, H_{p2}, H_c, \lambda) = \sum_{j=H_{p1}}^{H_{p2}} \left(w\left(k+j\right) - \hat{y}\left(k+j\right)\right)^2 + \lambda \sum_{j=1}^{H_c} \Delta u^2(k+j-1)\,. \tag{12}$$

Here, $\hat{y}(k+j)$ denotes the predicted process output, H_{p1} is the minimum costing horizon, H_{p2} is the maximum costing or prediction horizon, H_c is the control horizon, and λ is a weighting coefficient.

The predicted outputs are calculated by using the following approximate step-response model:

$$\hat{\mathbf{y}} = \mathbf{G}\mathbf{\Delta}\overline{\mathbf{u}} + \mathbf{p} \tag{13}$$

where $\hat{\mathbf{y}} = [\hat{y}(k+H_{p1}), \ldots, \hat{y}(k+H_{p2})]$ is the vector of the predicted outputs, $\mathbf{G}$ denotes the dynamic matrix builds up from the g_i step-response coefficients [3] generated from the extracted linear model, $\mathbf{\Delta}\overline{\mathbf{u}} = [\Delta u(k), \ldots, \Delta u(k+H_c)]$ is the vector of the calculated control sequence on the H_c control horizon, and the p_j elements of $\mathbf{p} = [p_1, p_2, \ldots, p_{H_{p_2}}]$ are the estimated response of the system at the $k+j$th step assuming that the future control signal remains constant. This free response can be obtained by using the fuzzy model or from the extracted linear model.

To cope with the model-plant mismatch and unmeasured disturbances, the MPC can implemented within an internal model control (IMC) scheme [5], see Figure 3.

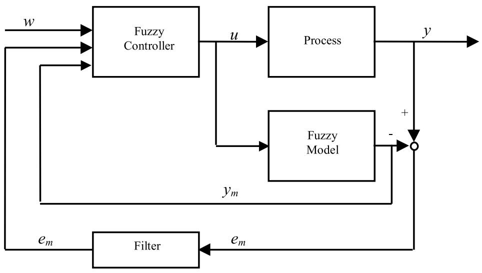


Figure 3. *Implementation of the MPC in the IMC structure.*

As the MPC can operate in IMC and non-IMC framework, the free response can be generated form the nonlinear or the extracted linear model, the linear model can be extracted by LPV interpretation or linearization, and the applied model can be identified by local or global identification, 16 different possible controllers can be designed. In order to compare the control performance of these configurations, the mean square of the control error is used as performance index. The prediction and the control horizons are selected to be $H_{p1} = 1,\ H_{p2} = 5$, and $H_c = 1$. The move suppression coefficient was set to $\lambda = 0$.

Because the process and its model is a Hammerstein type (the antecedent fuzzy sets are defined on the control signal), the free run of the nonlinear model is identical to the free run of the linear model extracted by the linearization of the model. Moreover, as the globally identified model perfectly describes the system, the IMC scheme does not have any meaning in this case. Hence, the original 16 variation of the control configurations have reduced to six cases. The control task and the performance of the globally identified, Lyapunov-linearization based MPC is shown in Figure 4. The control configurations and the results are shown in Table 1.

From these experiments, it has turned out, only models obtained by local estimation can be interpreted locally and applied in LPV based control design. These models, have to be used in IMC scheme to compensate the bad steady-state representation of the model. Models obtained by global identification do not lend themselves to local interpretation and thus cannot be used in LPV schemes. Such a model is only useful for prediction and/or when local parameters are obtained by Taylor-series linearization. The performance of the local identified and LPV interpreted model in the MPC placed in IMC scheme is comparable to the performance of the globally identified, Lyapunov-linearization based model. Hence, the use of this LPV configuration is advantageous, because the LPV interpretation is more transparent than the global one, and this transparency is the main advantage of using fuzzy models in model based control.

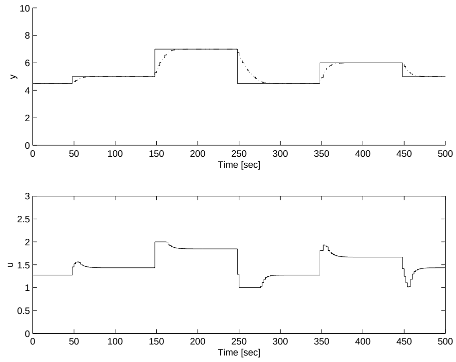


Figure 4. *Control performance of the globally identified, Lyapunov-linearization based MPC.*

Table 1. *MSE control performances of the various control configurations.*

Identification Method	Extraction Method	IMC	MSE
Global	Global	-	0.14
Global	LPV	-	not stable
Local	Global	no	0.206
Local	LPV	no	0.174
Local	Global	yes	0.185
Local	LPV	yes	0.147

5 Conclusions

An analysis of the application of Takagi–Sugeno fuzzy models in model based control has been presented. As the predictive controller can operate in IMC and non-IMC framework, the linear model can be extracted by LPV interpretation or Taylor linearization, and the applied model can be identified by local or global identification, the fuzzy model can be implemented in several different control configurations. The paper presented some guidelines and analysis about which configuration can be used for a locally or globally identified fuzzy models.

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